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A GENERAL ANALYSIS OF MOISTURE MIGRATION CAUSED BY TEMPERATURE DIFFERENCES IN AN UNSATURATED POROUS MEDIUM

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Professor Edmond A. Brun was interested in heat and mass transfer in porous media, as one of his wide-ranging areas of concern. A large number of papers, reports, and theses published by his students on this topic was initiated and influenced by him. To one paper he gave his name: E. Brun and B. Le Furo, Application of similitude in the mechanics of dispersed media; in *Problems of Hydrodynamics and Continuum Mechanics, S.I.A.M.*, 63–76 (1969) (in English).

The present contribution is dedicated in his memory.

Abstract — An analysis was performed which describes the moisture migration in a slab of an unsaturated porous material for the condition that the temperature of one surface is suddenly increased to a higher value whereas the temperature of the other surface is maintained constant. The two surfaces are assumed impermeable to mass flow. The thermodynamic and transport properties occurring in the equations describing the temperature and moisture transport are assumed constant. In this way, very general relations are obtained for the moisture field as it changes in time. Before dry-out, a dimensionless parameter describing the moisture field is a function of equivalent Luikov and Fourier numbers only. The dependence on the Luikov number becomes noticeable solely for the early periods and for small values of the Luikov number. After the onset of dry-out, the moisture ratio depends in addition on the thermal mass diffusion coefficient. The results of the analysis presented in the Figs 1–7, should describe the actual moisture migration with good accuracy where the variation of the properties is small in the range of the independent variables occurring in a specific situation and they should be useful as a first approximation for other situations as well.

NOMENCLATURE

c,	specific heat;
D,	vapor diffusion coefficient;
D^*,	see equation (12);
f,	porosity;
h,	enthalpy;
h_{lv},	heat of vaporization;
j,	mass flux;
k,	thermal conductivity;
K,	moisture diffusion coefficient;
L,	thickness of slab;
$\dot{m}$,	flux of liquid and vapor per unit time and area;
M,	molecular weight;
R,	universal gas constant;
t,	temperature;
T,	absolute temperature;
w,	vapor mass fraction;
W,	moisture content (mass of liquid per mass of dry soil);
x,	coordinate;
α,	thermal diffusivity;
ρ,	density;
ϕ,	potential;
τ,	time.

Subscripts

a,	air;
d,	dry soil;
i,	initial;
l,	liquid;
L,	at $x = L$;
0,	at $x = 0$;
s,	soil;
v,	vapor;
∞,	at time $\tau \to \infty$.

Dimensionless parameters

$\frac{K}{\alpha_s}$,	Luikov number;
$\frac{\alpha_s \tau}{L^2}$,	Fourier number;
$\frac{D^*(t_0 - t_i)}{KW_i}$,	thermal mass diffusion parameter;
$\frac{K\tau}{L^2}$,	moisture transfer Fourier number.

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INTRODUCTION

COMBINED heat and mass transfer with phase change in a porous medium is a process which occurs frequently in nature as well as in various engineering endeavors. Drying and humidification in chemical processes and in climate control or movement of moisture in the soil are examples. Such processes have, therefore, been widely studied by agricultural, civil, chemical, and petroleum engineers.

An analysis of these processes is complicated by various factors. The structure of the solid matrix varies widely in shape. It may, for instance, be composed of cells, fibres or grains. There is, in general, a distribution of void sizes and the structures may also be locally irregular. Energy transport in such a medium occurs by conduction in all of the phases as well as by convection with those phases which are able to move. Mass transport occurs within the voids of the medium. In an unsaturated state these voids are partially filled with a liquid, whereas the rest of the voids contains some gas. Most frequently, the liquid is water and the gas is air. We will, therefore, in the following refer to the liquid as water and to the gas as air. Evaporation or condensation occurs at the interface between the water and the air so that the air is mixed with water vapor. A flow of the mixture of air and vapor may be caused by external forces, for instance, by an imposed pressure difference. The vapor will also move relative to the gas by diffusion from regions where the partial pressure of the vapor is higher to those where it is lower. The partial pressure of vapor at the interface to the liquid is determined by the sorption isotherm, which makes it dependent on moisture content as well as on temperature. The saturation pressure is also different on a curved surface from that on a plane one and is influenced by the presence of air. The flow of liquid is caused by external forces, like imposed pressure differences, gravity, and internal forces, like capillary, intermolecular and osmotic forces.

A detailed study of the transport processes occurring within the solid matrix and in the voids is, therefore, very complicated even for a regularly shaped matrix and is impossible for the irregular void configurations existing in general in porous media. The normal approach in an analysis is, therefore, to consider the media involved as continua. The energy and mass fluxes then have to be described by constitutive equations. The various driving forces involved make the constitutive equations quite complicated. The difficulty in handling them is aggravated by the fact that the transport properties involved vary strongly with the structure of the porous medium, with moisture content, and with temperature. An extensive literature exists which endeavors to formulate the transport processes mathematically. Early work was done in the Soviet Union [1] with important basic contribution by Luikov [2]. A wide range of transport processes in porous media was also studied simultaneously in Germany by Krischer [3]. More recent work is referenced in the paper by Eckert and Pfender [4].

Numerous solutions of the transport equations are also found in the literature. Only some can be listed here [5–11]. The selection is restricted to the problem which will be discussed later in this paper. The referenced solutions consider certain types of media and boundary conditions and provide valuable information for the specific situation. It is, however, difficult to obtain from them a general understanding which can be used as a guide for conditions for which no specific solutions are available.

This is a situation similar to the one which existed in the field of heat transfer at the beginning of this century. A considerable number of relations obtained from experiments were at that time available for specific fluids, temperatures and boundary conditions. A systematic understanding of heat transfer processes, however, was obtained only when W. Nusselt introduced an idealized model of a constant property fluid. In this way he derived through dimensional analysis relations expressing Nusselt numbers as functions of Reynolds and Prandtl numbers, relations which describe heat transfer processes in a very general way. Such relations have been accepted by the engineering community and are still used widely today. They have in the meantime been supplemented by specific more accurate information which includes the variation of the properties involved.

It is felt that such general relations describing heat and mass transfer for certain classes of porous media would be valuable and that they can be obtained by an approach similar to the one mentioned above. Such relations can then be applied to situations where the actual variation of the properties is small in the range of independent variables occurring in the specific situation and are useful as a first approximation for other situations as well.

The present paper uses this approach for a class of heat and mass transfer processes characterized by the following specifications. The effect of gravity is negligible and there is no mass flow through the boundaries of the medium. The porous matrix is of such a nature that the saturation pressure at the interface between liquid and vapor is a function of temperature only. This is, for instance, the case in a granular material when only a small number of pores have sizes smaller than 1 μm and as long as the moisture content is not too small [4]. The pore size is, on the other hand, sufficiently small so that the vapor pressure throughout any cross-section of the vapor–gas passages departs from the saturation pressure at the interface by a negligible amount only. The medium is also postulated to be macroscopically homogeneous and not to swell with changing moisture content.

TRANSPORT EQUATIONS

The restrictions defining the class of media for this study have been mentioned before. The following restrictions will be introduced for the condition under which the transport processes occur:

1. No mass flow passes the envelope of the porous medium.
2. The vapor pressure p_v is small compared to the total pressure p which, therefore, can be approximated by the air pressure p_a.
3. Temperature differences imposed on the medium are small so that the total pressure (and the air pressure) can be considered constant during the transport processes. The influence of a slight movement of the air within the voids of the material will therefore be neglected and the diffusion of vapor in the voids will be described by Fick's equation.

The conservation equation for energy can then be written in the following form

$$\rho_d c_s \frac{\partial t}{\partial \tau} = \nabla(k\nabla t) + (h_{lv} + h_l) j_v + h_l j_l. \qquad (1)$$

The various symbols in the equation are defined in the list of nomenclature. The term on the left hand side describes energy storage in an infinitely small control volume. c_s denotes the specific heat of the moist medium per unit dry mass and ρ_d denotes the dry density. The first term on the right hand side describes heat flow by conduction. The thermal conductivity k has to include heat conduction in the liquid, vapor, and air as well as in the solid material. The second term on the right hand side describes enthalpy transport with the vapor. j_v denotes the vapor mass flux and the enthalpy of the vapor is expressed as the sum of the heat of evaporation h_{lv} and of the sensible heat h_l of the liquid. The third term on the right hand side describes the enthalpy transport with the liquid. j_l denotes the mass flux of the liquid.

No net mass flux occurs through the boundaries of the porous material; accordingly $j_l + j_v = 0$. The same condition holds in the interior of the medium for steady state. During the transient period there will be a small net mass flux in the interior. Its influence on the enthalpy transport will be neglected with the justification that the heat of vaporization h_{lv} is large compared to the sensible heat of the liquid h_l. Equation (1) then transforms to the following equation when in addition the vapor flux j_v is expressed by Fick's equation.

$$\rho_d c_s \frac{\partial t}{\partial \tau} = \nabla(k\nabla t) + \nabla(\rho_a D_s h_{lv} \nabla w). \qquad (2)$$

The driving potential in the diffusion equation is expressed as a gradient of the mass fraction w of the diffusing vapor. D_s describes the mass diffusion coefficient of vapor in the porous matrix. It has to account for the fact that only part of the cross-section of the medium is available for vapor diffusion and also for the tortuosity of the diffusion path. The total density in the diffusion equation has been approximated by the air density ρ_a. The cross sections of the passages in the porous medium are in general quite small, justifying the assumption that the partial pressure of vapor at any location is equal to the saturation pressure at the local temperature. This pressure is in porous media a function of temperature t and moisture content W and is given by the sorption isotherm. At a prescribed constant pressure such a relation holds also for the vapor mass fraction w

$$w = f(W, t). \qquad (3)$$

The operator ∇w can therefore be expressed in the following way

$$\nabla w = \frac{\partial w}{\partial t} \nabla t + \frac{\partial w}{\partial W} \nabla W. \qquad (4)$$

The dependence of the saturation pressure p_v and the saturation mass fraction w on moisture content W has various causes. The equilibrium saturation pressure of a liquid in a dispersed phase is different from that in a bulk liquid. This effect can be treated thermodynamically. Equation (70) in [11] describes the relation between the saturation pressure p_v and the saturation pressure p_{v0} of vapor in contact with its bulk liquid phase through a plane interface. The first term on the right hand of this equation describes the fact that the saturation pressure in a vapor–air mixture is different from the saturation pressure in pure vapor. This effect, however, is negligibly small as long as the total pressure of the mixture is not too far away from atmospheric pressure. The most important of the effects is that of capillarity described by the second term. In a grainy matrix the potential Φ in the referenced equation has then to be replaced by $-(\sigma/\rho_l r_m)$ where σ is the surface tension, r_m is the mean radius of curvature, and ρ_l is the density of the liquid. On a concave surface, the difference of the saturation pressure on a plane surface and on a concave surface is positive and becomes larger with decreasing radius of curvature of the surface. The difference is 10% when the mean radius of curvature of the surface is 10^{-8} μm. Such small curvatures are expected at very fine porous material or at very low moisture content only. Adsorption effects become important in very thin liquid layers comprising only a few molecules. The liquid behavior differs in such a layer from a liquid in bulk when the layer is 10 molecules or less thick. This means that layers of order 10^{-9} μm are involved, which again indicates that the moisture content has to be very small. Sorption isotherms which describe these effects, for a number of materials are presented on pages 52–62 of [3]. One finds there that a 10% reduction of the saturation pressure occurs, for instance, for concrete at a moisture content $W = 0.02$ and that for brick the corresponding moisture content is 0.002. The second term in equation (4) will, therefore, be dropped for the class of porous materials considered in this paper. The energy equation (2) can then be transformed to

$$\rho_d c_s \frac{\partial t}{\partial \tau} = \nabla(k_s \nabla t) \qquad (5)$$

in which the soil conductivity k_s is given by the equation

$$k_s = k + \rho_a D_s h_{lv} \frac{\partial w}{\partial t}. \qquad (6)$$

This conductivity is the one which is obtained by conductivity measurements in a porous medium [4].

The conservation equation for moisture can be written in the following form

$$\rho_d \frac{\partial W}{\partial \tau} = \nabla(\rho_a D_s \nabla w) + \rho_l \nabla(K_\phi \nabla \phi). \qquad (7)$$

The equation expresses the fact that the vapor transported by diffusion into a volume element of soil and the liquid transported by various forces has to be stored and increases the moisture content in the element.

The left hand side of the equation should contain a term describing the storage of vapor in addition to the one describing the storage of liquid. The vapor storage, however, is extremely small compared with the liquid storage and will be neglected. An upper bound for the vapor mass stored per unit volume is $[f\rho_a(\partial w/\partial t)(\partial t/\partial \tau)]$. The moisture transport in liquid form described by the second term on the right hand side is written as a function of the gradient of the suction potential ϕ. The moisture diffusivity K_ϕ depends on the parameters describing the porous matrix, on the moisture content, and on the temperature. For a specific homogeneous soil it is therefore a function of W and t.

$$\Phi = f(W, t). \qquad (8)$$

The dependence on temperature is in general considerably smaller than the dependence on moisture content W and will be neglected. The conservation equation then takes on the form

$$\frac{\partial W}{\partial \tau} = \frac{1}{\rho_d} \nabla\left(\rho_a D_s \frac{\partial w}{\partial t} \nabla t\right) + \nabla(K \nabla W) \qquad (9)$$

into which a new moisture diffusivity K has been introduced defined by the equation

$$K = \frac{\rho_l}{\rho_d} K_\phi \frac{\partial \phi}{\partial W}. \qquad (10)$$

CONSTANT PROPERTY SOLUTIONS

Slab with a stepwise temperature variation

A slab of thickness L will be considered as a specific example. The temperature and moisture content are prescribed as initially uniform and the temperature of one surface is suddenly increased and kept constant whereas the temperature of the other surface is maintained at the original value. The energy equation (5) takes on the form when the properties are considered constant

$$\frac{\partial t}{\partial \tau} = \alpha_s \frac{\partial^2 t}{\partial x^2}. \qquad (11)$$

With the definition of a thermal mass diffusion coefficient D^*, which is also considered constant, defined by

$$D^* = \frac{\rho_a}{\rho_d} D_s \frac{\partial w}{\partial t} = \frac{M_v D_s}{\rho_s R T} \frac{\partial p_v}{\partial t} \qquad (12)$$

the moisture conservation equation reads

$$\frac{\partial W}{\partial \tau} = D^* \frac{\partial^2 t}{\partial x^2} + K \frac{\partial^2 W}{\partial x^2}. \qquad (13)$$

The boundary conditions for the problem are

$$\text{at} \quad \tau = 0: t = t_i, \ W = W_i \qquad (14)$$

$$\text{at} \quad \tau > 0, \ x = 0: \ t = t_0, \ \dot{m} = 0,$$

$$x = L: \ t = t_i, \ \dot{m} = 0$$

where the specific mass flux $\dot{m}$ is given by

$$\dot{m} = -D^* \frac{\partial t}{\partial x} - K \frac{\partial W}{\partial x}. \qquad (15)$$

It is evident that the energy equation (11) with its boundary conditions is independent of the moisture field. The equation and its boundary conditions will now be made dimensionless by the following change of variables

$$\mathrm{x} = \frac{x}{L}, \ \tau_h = \frac{\alpha_s \tau}{L^2}, \ \mathrm{t} = \frac{t - t_i}{t_0 - t_i}. \qquad (16)$$

It is noted that no value for a characteristic time is prescribed to the problem. Such a time has, therefore, to be formed with a proper combination of other prescribed parameters. The energy equation now takes on the form

$$\frac{\partial \mathrm{t}}{\partial \tau_h} = \frac{\partial^2 \mathrm{t}}{\partial \mathrm{x}^2}. \qquad (17)$$

The dimensionless boundary conditions are:

$$\tau_h = 0: \mathrm{t} = 0$$

$$\tau_h > 0, \mathrm{x} = 0: \ \mathrm{t} = 1, \mathrm{x} = 1: \ \mathrm{t} = 0. \qquad (18)$$

It is observed that neither in the differential equation (17) nor in the boundary conditions (18) an arbitrary constant appears. The solution of the temperature field will, therefore, have the form

$$\frac{t - t_i}{t_0 - t_i} = f\left(\frac{\alpha_s \tau}{L^2}, \frac{x}{L}\right). \qquad (19)$$

It is, of course, a well-known solution of an unsteady heat conduction problem. The parameter $\alpha_s \tau / L^2$ is referred to as Fourier number.

The moisture conservation equation (13) is made dimensionless with the following new variables

$$\mathbf{x} = \frac{x}{L},\quad \tau_m = \frac{K\tau}{L^2},\quad \Delta\mathbf{W} = \frac{K(W-W_i)}{D^*(t_0-t_i)}. \qquad (20)$$

It is observed that only differentials of the moisture content appear in equation (13). The moisture content W can then be interpreted as the difference between the moisture content W and its original value W_i. In this way no prescribed moisture content is available and the dimensionless parameter $\Delta\mathbf{W}$ has to be formed with a reference moisture content formed by a proper combination of the prescribed values. This is done in the last equation (20). The moisture conservation equation has the form

$$\frac{\partial(\Delta\mathbf{W})}{\partial\tau} = \frac{\partial^2\mathbf{t}}{\partial\mathbf{x}^2} + \frac{\partial^2(\Delta\mathbf{W})}{\partial\mathbf{x}^2} \qquad (21)$$

with the boundary conditions

$$\tau = 0:\ \mathbf{W} = 0$$

$$\tau > 0,\ \mathbf{x} = 0 \quad \text{and} \quad \mathbf{x} = 1:\ \frac{\partial\mathbf{t}}{\partial\mathbf{x}} + \frac{\partial(\Delta\mathbf{W})}{\partial\mathbf{x}} = 0. \qquad (22)$$

The temperature now appears in the differential equation and in one of the boundary conditions and the solution of equation (21) thus depends on the temperature field expressed by equation (19). The solution will therefore have the form

$$\frac{K(W-W_i)}{D^*(t_0-t_i)} = f\left(\frac{K\tau}{L^2}, \frac{K}{\alpha_s}, \frac{x}{L}\right). \qquad (23)$$

The dimensionless parameter $K\tau/L^2$ has the same form as the Fourier number in equation (19) and may be called moisture transfer Fourier number, the parameter K/α_s is referred to in the literature as Luikov number.

Solutions of the equations (17) and (21) with the boundary conditions (18) and (22) were obtained by computer analysis. The solution procedure is described in the appendix. The solutions are presented in Figs. 1 and 2.

Figure 1 shows the dimensionless moisture parameter $\Delta\mathbf{W}$ as a function of the dimensionless distance x/L. The parameter on the curves is the dimensionless time $K\tau/L^2$ counted from the start of the temperature change on the surface $x = 0$. It may be called moisture transfer Fourier number. The full lines present the moisture fields for the parameter, $\alpha_s/K = 250$. The inverse of this parameter is in mass transfer processes referred to as Luikov number. The dash–dotted lines are for the parameter $\alpha_s/K = 10$. Two lines are also shown for $\alpha_s/K = 100$. It can be observed that the Luikov number exerts an influence on the development of the moisture field for increasingly shorter time periods as the value of the Luikov number decreases. This is so because the development of the moisture field lags behind the development of the temperature field, especially at large values of α_s/K. The term $\partial^2\mathbf{t}/\partial\mathbf{x}^2$ in equation (21) vanishes when the temperature profile becomes linear. The fact that the mass transfer equation assumes the same form as the unsteady heat conduction equation for small values of the Luikov number has also been pointed out in [9].

The way in which the moisture field develops can be understood from the following physical considerations. The transport of vapor by diffusion is rigidly coupled to the development of the temperature field according to equation (4), without the second right hand term. The vapor flux is constant along the distance x in the slab when the temperature drop is linear and this is the case for the steady temperature profile. No vapor has, therefore, to be created or absorbed in the interior of the slab and the moisture

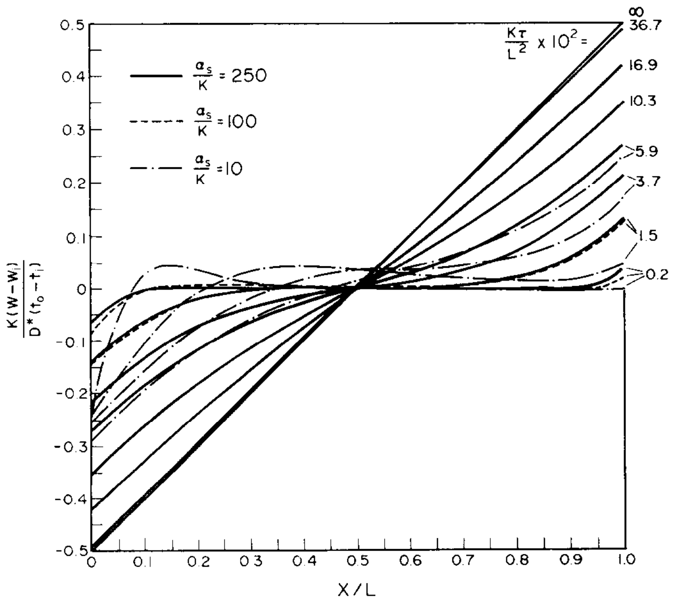

FIG. 1. Local moisture content parameter in a porous slab as a function of the moisture transfer Fourier number for three values of the reciprocal Luikov number.

content can remain at its original value. Only at the two surfaces the situation is different. The vapor moving away from the heated surface ($x = 0$) has to be replaced by evaporation of the liquid. Correspondingly, the moisture content drops in the neighborhood of this surface. The inverse situation is found at the cooled surface $x = L$. Mathematically the situation described above is expressed by the fact that the driving force term vanishes in equations (13) and (21).

The situation is different when the temperature field is not linear as it occurs during the developing period of the temperature field. In this case, the mass flux of vapor by diffusion decreases along the distance x and vapor has to be absorbed by condensation. This then increases the moisture content in the interior of the slab even before the change initiated at the boundaries reaches that location. This effect can clearly be seen in the figure. Mathematically it is indicated by the fact that a force term depending on temperature appears now in equations (13) and (21).

Figure 2 is obtained from Fig. 1 by plotting the dimensionless moisture concentration parameter $\Delta\mathbf{W}$ at $\mathbf{x} = 0$ over the dimensionless time. The fact that part of the vapor flux is created by evaporation in the interior of the slab is obviously the explanation for the change in the concentration parameter with varying Luikov number in the early time zone.

The solution presented in Figs 1 and 2 has physical significance for positive values of the moisture concentration W only. A limiting condition is, therefore, that the moisture concentration W reaches the value zero at $x = 0$. This limiting condition for the initial moisture content W_i at which dryout is obtained at $x = 0$ and time τ can be obtained from equation (23) by setting $W = 0$ at $x = 0$. The resulting equation has the form

$$\frac{KW_{id}}{D^*(t_0 - t_i)} = f\left(\frac{K\tau}{L^2}, \frac{K}{\alpha_s}\right). \tag{24}$$

Figure 2 actually presents this functional relation when the parameter $\Delta\mathbf{W}_0$ on the ordinate is interpreted as $-KW_{id}/[D^*(t_0 - t_i)]$. For any value $\Delta\mathbf{W}_0$ one can calculate the W_i leading to $W = 0$ by the equation

$$W_{id} = -\Delta\mathbf{W}_0 \frac{D^*(t_0 - t_i)}{K}. \tag{25}$$

The time at which $W = 0$ is reached can then be read on the abscissa.

Dryout of a slab

The solutions obtained in the previous paragraph lose their validity beyond the time when the surface $x = 0$ has reached the moisture content $W = 0$. From then on a dry layer develops adjoining this surface which increases with increasing time. The developing moisture field during this period is considered in this section.

Dryout starts at a time when the temperature field has already reached steady state unless the ratio α_s/K has a very small value. In this section, it is, therefore, assumed that the temperature field is developed. The equations (26) and (27) below describe the moisture field and the boundary conditions. The problem is analogous to heat conduction with a moving boundary.

$$\frac{\partial W}{\partial \tau} = K\frac{\partial^2 W}{\partial x^2}. \tag{26}$$

With τ_d indicating the time at which $W = 0$ at $x = 0$, the starting condition is

$$\tau = \tau_d, \quad W = W_d$$

W_d denotes the moisture content at the beginning of dryout taken from Fig. 1.

The boundary conditions are

$$\tau > \tau_d,\ x = \xi:\ W = 0, \quad W\frac{d\xi}{d\tau} = -D^*\frac{\partial t}{\partial x} - K\frac{\partial W}{\partial x} \tag{27}$$

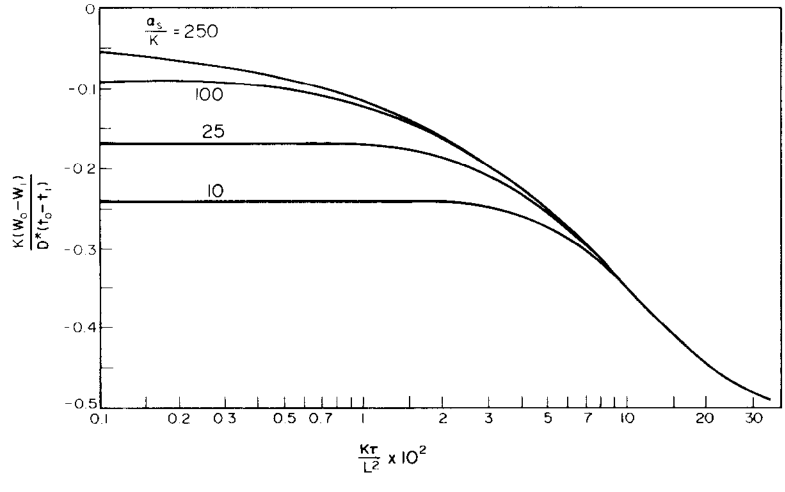

FIG. 2. Moisture content parameter at the hot surface of the porous slab plotted over the moisture transfer Fourier number for four values of the reciprocal Luikov number.

$$\tau > \tau_d,\ x = L:\ D^*\frac{\partial t}{\partial x} + K\frac{\partial W}{\partial x} = 0.$$

The thickness of the dry layer is called ξ.

The equations are made dimensionless with the following variables

$$\mathbf{x} = \frac{x}{L},\quad \boldsymbol{\xi} = \frac{\xi}{L},\quad \boldsymbol{\tau} = \frac{K\tau}{L^2},\quad \mathbf{W} = \frac{W}{W_i}. \tag{28}$$

This results in the dimensionless mass conservation equation

$$\frac{\partial \mathbf{W}}{\partial \boldsymbol{\tau}} = \frac{\partial^2 \mathbf{W}}{\partial \mathbf{x}^2} \tag{29}$$

and the dimensionless boundary conditions

$$\mathbf{x} = \boldsymbol{\xi}:\ \mathbf{W}\frac{\mathrm{d}\boldsymbol{\xi}}{\mathrm{d}\boldsymbol{\tau}} = -\frac{D^*(t_0 - t_i)}{KW_i}\frac{\partial \mathbf{t}}{\partial \mathbf{x}} - \frac{\partial \mathbf{W}}{\partial \mathbf{x}}$$

$$\mathbf{x} = 1:\ \frac{D^*(t_0 - t_i)}{KW_i}\frac{\partial \mathbf{t}}{\partial \mathbf{x}} + \frac{\partial \mathbf{W}}{\partial \mathbf{x}} = 0 \tag{30}$$

in which the dimensionless temperature gradient is

$$\frac{\partial \mathbf{t}}{\partial \mathbf{x}} = 1. \tag{31}$$

There are now two dependent variables $\boldsymbol{\xi}$ and $\mathbf{W}$ and it is seen from equations (29) and (30) that the solutions will have the form

$$\frac{\xi}{L} = f\left(\frac{D^*(t_0 - t_i)}{KW_i}, \frac{K\tau}{L^2}\right) \tag{32}$$

$$\frac{W}{W_i} = f\left(\frac{D^*(t_0 - t_i)}{KW_i}, \frac{K\tau}{L^2}, \frac{x}{L}\right). \tag{33}$$

The results of a computer calculation are presented in Figs. 3–6 for the values 2.5, 5, 7.5 and 10 of the thermal mass diffusion parameter $D^*(t_0 - t_i)/(KW_i)$. The computation procedure is again described in the appendix.

It can be observed that the moisture ratio W/W_i decreases near the hot surface of the slab and increases near the cold surface. The curve for $K\tau/L^2 \times 10^2 = 0.05$ belongs to a parameter $\alpha_s/K = 250$. For the other curves, the influence of this parameter disappears. After a certain time, described by an equivalent Fourier number $K\tau_d/L^2$, the moisture at the hot surfaces reaches the value zero. At that moment dry-out starts and the dried out region expands into the interior of the slab. An asymptotic 'steady' state situation is finally reached in which the moisture content increases linearly with increasing depth of the slab. The depth to which dry-out penetrates increases with increasing value of the thermal mass diffusion parameter $D^*\ (t_0 - t_i)/KW_i$ as seen from a comparison of the Figs 3–6. The total moisture content in the slab has to remain constant. Accordingly, the local moisture content at the cool surface increases with increasing value of the thermal mass diffusion parameter. The location of the dry-out front as a function of the dimensionless time, described by the moisture transfer Fourier number $K\tau/L^2$, is presented in Fig. 7.

The local moisture content for the steady state can be calculated in a simple way. The original total moisture content in the slab, expressed in the dimensionless parameters shown in Figs. 3–6, has the area 1.

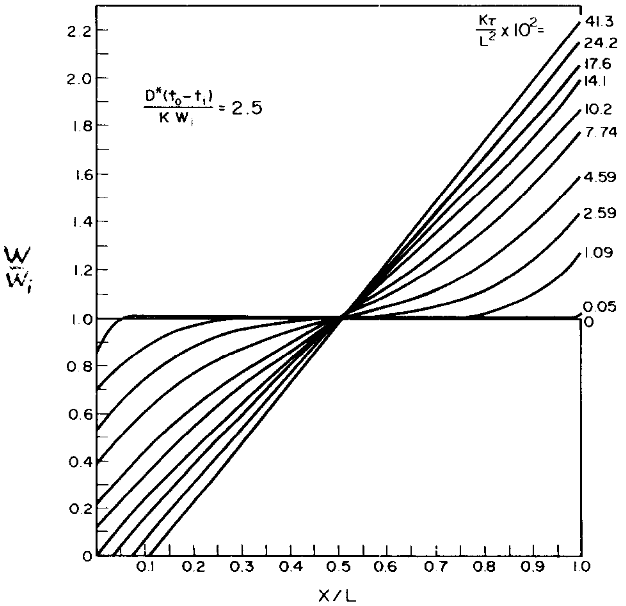

FIG. 3. Local moisture ratio in a slab under dry-out conditions with the moisture transfer Fourier number as parameter. The thermal mass diffusion parameter has the value 2.5.

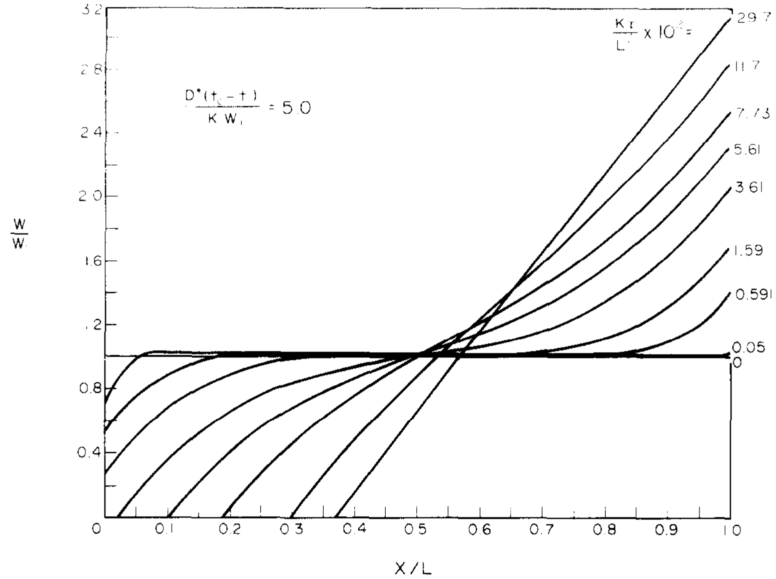

FIG. 4. Local moisture ratio in a slab under dry-out conditions with the moisture transfer Fourier number as parameter. The thermal diffusion parameter has the value 5.

The triangle in the figure formed with the steady moisture ratio must have the same area. This leads to the equation

$$\left(1 - \frac{\xi_\infty}{L}\right)\frac{W_{L\infty}}{W_i} = 2. \tag{34}$$

For steady state, $\dot{m}$ has to be zero and equation (15) changes to

$$-\frac{\mathrm{d}t}{\mathrm{d}W} = \frac{K}{D^*}. \tag{35}$$

From the linear character of the temperature and the moisture profiles one obtains

$$\frac{(t_0 - t_i)\left(1 - \frac{\xi}{L}\right)}{W_{i\infty}} = \frac{K}{D^*}. \tag{36}$$

With the parameter m defined by the equation

$$m = \frac{D^*(t_0 - t_i)}{kW_i} \tag{37}$$

one obtains

$$\frac{\frac{W_{L\infty}}{W_i}}{1 - \frac{\xi_\infty}{L}} = m. \tag{38}$$

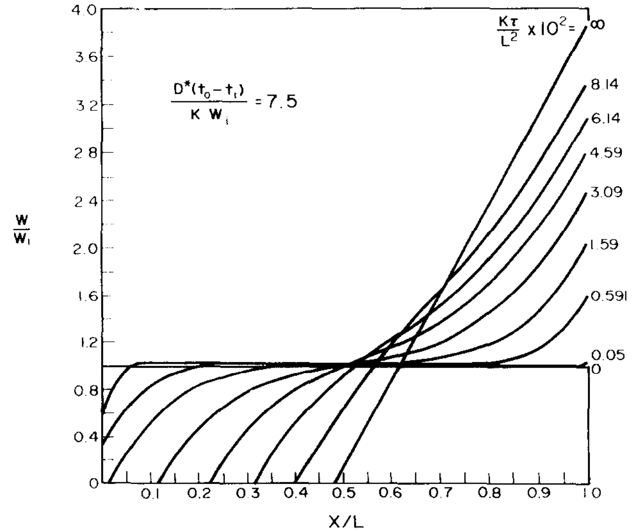

FIG. 5. Local moisture ratio in a slab under dry-out conditions with the moisture transfer Fourier number as parameter. The thermal diffusion parameter has the value 7.5.

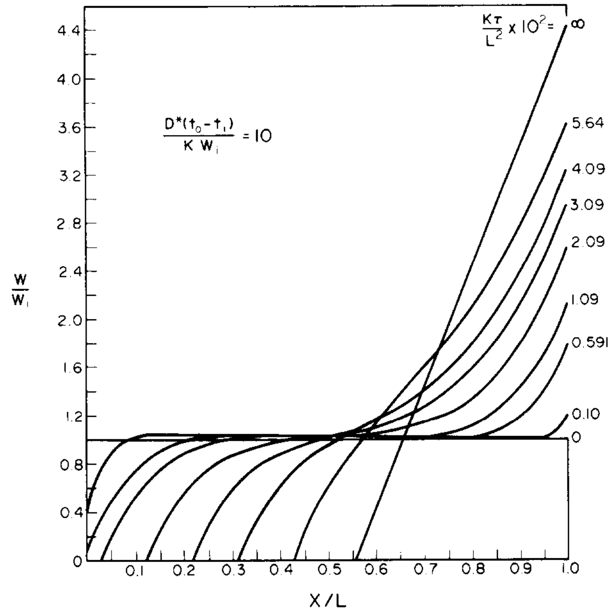

FIG. 6. Local moisture ratio in a slab under dry-out conditions with the moisture transfer Fourier number as parameter. The thermal diffusion parameter has the value 10.

The equations (34) and (38) can be solved for the location ξ_∞ of the dry-out front and for the moisture content $W_{1\infty}$ at the cold surface. One obtains

$$\frac{\xi_\infty}{L} = 1 - \sqrt{\frac{2}{m}} \tag{39}$$

$$\frac{W_{L\infty}}{W_i} = \sqrt{2m}. \tag{40}$$

Numerical examples

One of the dry-out curves in the Figs. 3–6 will be compared with a dry-out curve measured by Krischer and Rohnalter and reproduced on page 220 [3]. The measurements were made in the way that a horizontal tube 50 cm long was filled with moist sand with an average grain diameter of 0.2 mm. The two ends of the tubes were sealed and kept at constant temperatures of 70°C and 20°C for a period of 5 months. The originally uniform moisture content was rearranged by the temperature field and steady state was established by the end of that time period. The tube was opened and the local moisture content was measured along the axis of the tube. A sketch of the tube and two moisture profiles are reproduced in Fig. 8. It may be observed that dry-out occurred when the original moisture content was $W_i = 0.025$. This curve will now be compared with one of the calculated steady state moisture distributions.

The ratio of the moisture content W_L to the original moisture content has the value 6.8 according to Fig. 8. Equation (40) results with this ratio in the parameter

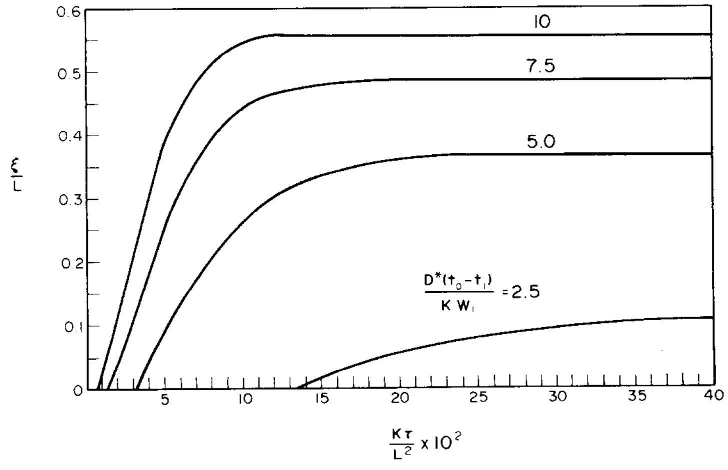

FIG. 7. Location of the dry-out front in a heated porous slab plotted over the mass transfer Fourier number for four values of the thermal mass diffusion parameter.

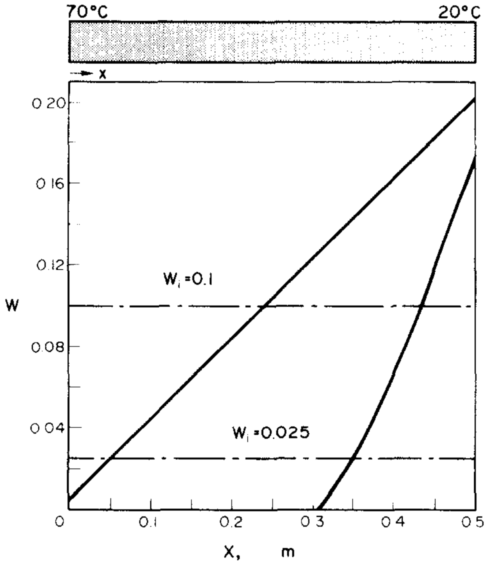

FIG. 8. Local rearrangement of the moisture content of an unsaturated sand after steady state is asymptotically attained.

$m = 23.1$. The tangent to the moisture curve shown as dashed line in Fig. 8 results in the value 0.31 for the parameter $1 - \xi/L$ and equation (39) gives an m value of 20.8 in fair agreement with the value obtained above. Assuming that the temperature gradient along the tube and the moisture gradient in the wet region of the tube are essentially constant, we can rewrite equation (35)

$$\frac{K}{D^*} = \frac{\Delta T}{\Delta W}. \tag{41}$$

The temperature drop in the moist region of the tube is

$$\Delta t = 50\frac{0.175}{0.5} = 17.5°\text{C} \tag{42}$$

and the moisture increase in this region is

$$\Delta W = 0.17. \tag{43}$$

With this one obtains from equation (41)

$$\frac{K}{D^*} = \frac{\Delta t}{\Delta W} = \frac{17.5}{0.17} = 113°\text{C}. \tag{44}$$

The vapor diffusivity through the porous material is estimated to be approximately 1/5 of the diffusivity of vapor in a bulk mixture of water vapor and air.

$$D_s = 4 \times 10^{-6}\,\text{m}^2/\text{s}. \tag{45}$$

From equation (12) one obtains the value

$$D^* = 0.954 \times 10^{-11}\,\text{m}^2/\text{s}\,°\text{C} \tag{46}$$

and equation (44) results in the moisture diffusivity

$$K = 1.185 \times 10^{-9}\,\text{m}^2/\text{s}. \tag{47}$$

With this parameter one can now determine the time at which steady state is approached under the experimental conditions using the moisture transfer Fourier number $K\tau/L^2$ from Fig. 7. This value is estimated to be

$$\frac{K\tau}{L^2} \approx 0.15. \tag{48}$$

The time at which steady state is approached is

$$\tau \approx \frac{0.05 \times 0.25}{1.185 \times 10^{-9}} \approx 1.05 \times 10^7\,\text{s} \approx 4\ \text{months}. \tag{49}$$

This time is in good agreement with the time established by Krischer and Rohnalter. The comparison in this section is taken to indicate that the constant property analysis provides a reasonable first approximation to dry-out processes.

Figure 8 shows that the measured gradient is smaller near the dry-out front than near the cool end of the pipe. Figure 6 shows the opposite behavior for transient conditions and any of the variations of the transport properties neglected in the analysis would also lead to a gradient of the moisture distribution curve which is larger in the neighborhood of the dry-out front than further away. No explanation for the opposite trend found in the experiments can be offered at present.

Figures 3–7 provide also the possibility to obtain the approximate time after which dry-out starts. An extrapolation of the curves which start with $W = 0$ at $x = 0$ results in an approximate Fourier number $K\tau_d/L^2 = 0.2$. An introduction of the same moisture diffusivity and length of the sample gives a time τ_d of approximately five days.

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APPENDIX

Solution methodology

The primary tool used in solving the foregoing equations is the Patankar–Spalding method [12]. This is a fully implicit finite difference scheme designed for two-dimensional parabolic type problems. The method has an extensive prior use for boundary layer problems and it is similarly being used for the present problem with an appropriate modification of the moving boundary. The solution is obtained by starting with known values at $\tau = 0$ and marching with time in the direction of increasing τ. For the case of no dry-out equations (17) and (21) with boundary conditions (18) and (22) were solved with 200 grid points in the region $0 < x \le 1$. For the numerical computation, the dimensionless time, τ_n was defined as $K\tau/L^2$ so that α_s/K appeared as a parameter in equation (17). In the τ direction, the grid encompassed about 5000 points when computation was continued until steady condition was reached. Aside from the accuracy tests involved with the step size studies, comparisons were made for the steady state condition with the analytical results. They agreed to within 0.03 %. For the case of dry-out, the problem is analogous to a heat conduction with a moving boundary. For this case, equations (29) and (31) were employed. The boundary condition at $x = \xi$ was used by treating the boundary as a porous wall with the mass flow out of it calculated from equation (30). The deployment pattern of the grid points is the same as for the case of no dry-out. The computed steady moisture profiles agree to within 0.01 % with the exact analytical results.

UNE ANALYSE GENERALE DE LA MIGRATION D'HUMIDITE CAUSEE PAR DES DIFFERENCES DE TEMPERATURE DANS UN MILIEU POREUX NON SATURE

Résumé — Une analyse décrit la migration d'humidité dans une plaque de matériau poreux non saturé lorsque la température d'une face est soudainement portée à une valeur plus élevée tandis que la température de l'autre face est maintenue constante. Les deux faces sont supposées imperméables au flux massique. Les propriétés thermodynamiques et de transport intervenant dans les équations qui décrivent la température et le transport d'humidité sont supposées constantes. Des relations très générales sont obtenues pour le champ d'humidité et son changement dans le temps. Avant l'assèchement, un paramètre sans dimension décrivant le champ de température est une fonction des nombres équivalents de Luikov et de Fourier. Après l'apparition de l'assèchement, le taux d'humidité dépend en plus du coefficient de diffusion couplée. Les résultats de l'analyse présentés dans les Figs. 1–7 décrivent la migration d'humidité avec une bonne précision quand la variation des propriétés est faible dans le domaine des variables indépendantes et ils peuvent être utilisés comme une première approximation dans les autres cas.

EINE ALLGEMEINE ANALYSE DER DAMPFWANDERUNG AUFGRUND VON TEMPERATURUNTERSCHIEDEN IN EINEM UNGESÄTTIGTEN PORÖSEN MEDIUM

Zusammenfassung—Es wurde eine Analyse durchgeführt, welche die Dampfwanderung beschreibt in einer Platte aus ungesättigtem porösem Material unter der Bedingung, daß die Temperatur der einen Seite plötzlich auf einen höheren Wert angehoben wird, während die Temperatur der anderen Oberfläche konstant bleibt. Für den Stofftransport werden die beiden Oberflächen als undurchdringbar angenommen. Die thermodynamischen und die Transportgrößen in den Gleichungen des Temperatur- und Dampftransports sind als konstant vorausgesetzt. Damit werden sehr allgemeine Beziehungen für das zeitlich veränderliche Dampffeld erhalten. Vor dem Einsetzen des dry-out beschreibt ein dimensionsloser Parameter das Dampffeld als eine Funktion einer äquivalenten Luikov- und Fourier-Zahl. Die Abhängigkeit von der Luikov-Zahl wird nur merklich für frühe Perioden und für kleine Werte der Luikov-Zahl. Nach dem Einsetzen des dry-out hängt das Dampfverhältnis zusätzlich von einem thermischen Massendiffusionskoeffizienten ab. Die Ergebnisse der Analyse, wie sie in Fig. 1–7 angegeben sind, beschreiben die eigentliche Dampfwanderung mit guter Genauigkeit, wenn die Änderung der Stoffwerte gering ist wie im Bereich unabhängiger Variabler die in speziellen Situationen auftreten, für andere Fälle aber sollten sie als nützliche erste Annäherungen dienen können.

ОБЩИЙ АНАЛИЗ ПЕРЕМЕЩЕНИЯ ВЛАГИ ПОД ДЕЙСТВИЕМ ПЕРЕПАДА ТЕМПЕРАТУР В НЕНАСЫЩЕННОЙ ПОРИСТОЙ СРЕДЕ

Аннотация — Проведен анализ перемещения влаги в плите из ненасыщенного пористого материала в случае, когда температура одной поверхности мгновенно возрастает, а другой поддерживается постоянной. Предполагается, что обе поверхности являются непроницаемыми для потока массы и что термодинамические и транспортные свойства в уравнениях тепло- и влагопереноса являются постоянными. Благодаря принятым допущениям получены весьма общие соотношения для описания изменения во времени содержания влаги. До наступления высыхания безразмерный параметр, описывающий распределение влагосодержания является функцией только чисел Лыкова и Фурье. Влияние числа Лыкова заметно только на начальной стадии и только в случае его малых значений. В режиме высыхания на относительное влагосодержание начинает также оказывать влияние коэффициент термодиффузии массы. Результаты анализа, представленные на рис. 1–7, могут с удовлетворительной точностью описывать действительную картину перемещения влаги при небольшом изменении характеристик процесса в области независимых переменных в одном частном случае, и их можно использовать в качестве первого приближения при описании процесса, протекающего в других условиях.